JOURNAL
OF
GEOMAGNETISM
AND
GEOELECTRICITY

VOL. II NO. 2

SOCIETY

OF

TERRESTRIAL MAGNETISM AND ELECTRICITY

Aug. 1950

KYOTO

Electro-Dynamical Behaviour of the Ionosphere Region Viewed from Geomagnetic Variations*

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Abstract

Dynamo-theoretical calculations of the longitudinal inequality of S_q-field (§2), the solar flare type variation (§5), S_D-field and magnetic bays (§§6 and 8) show a fairly good agreement with respective observed facts, not only in their mean states but also in their dynamic behaviours. Thus, it is concluded that the dynamo-action of the ionosphere can be responsible for various geomagnetic variations such as mentioned above as well as S_q and L.

Taking into account the screening effect of the ionosphere and the transient characteristics of electric current in the ionosphere, the integrated conductivity of the ionosphere is presumed to be 5×10^{-8} $\sim10^{-7}$ e.m.u., in place of 3×10^{-6} hitherto assumed, and the velocity of its lateral motion is estimated to be of order of magnitude 10^2 m/sec. Finally, some remarks on the dynamical characters of magnetic and ionospheric storms are mentioned from the above view point.

§1. Possible Relation of Ionosphere to Geomagnetic Variations.

It has been concluded by theoretical as well as observational studies that the dynamo-action must play the leading role in the electro-dynamical behaviour of the ionosphere region. The application of the interpretation of S_q- and L-variations in geomagnetic field has been developed by Stewart, Schuster and Chapman. The facts based on the observation of the ionosphere that the ionosphere is electrically conductive, and that it moves dynamically are in favour of this view. That is, its conductivity has been estimated from the data of various ionospheric observations such as electron number, collision frequency, the coefficients of recombination, attachment, and datachment with ions; while, the tidal motions of its height[1] and the lateral flow in its particular part[2] has been found out recently.

It is the object of our present study to construct a unified picture of the electrical and dynamical state of the ionosphere region, as consistent as possible in all kinds of geomagnetic and ionospheric observations.

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* Contribution from the Division of Geomagnetism and Geoelectricity, Geophysical Institute, Tokyo University. Series II, No. 10 (1950)

The fundamental equations and assumptions concerned here will be summarized in the followings.

The general equation describing electromagnetic field is given by

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\mu i, \tag{1}$$

where A, i, μ are vector potential, current density and magnetic permeability respectively, and c the light velocity. Current density, electric field intensity, E, magnetic induction, B, and scalar potential, ϕ, are related to vector potential in the following ways:

$$i = \sigma E, \tag{2}$$

$$E = -\frac{\partial A}{\partial t} - \mathrm{grad}\,\phi + [V\,\mathrm{rot}\,A], \tag{3}$$

$$B = \mathrm{rot}\,A, \tag{4}$$

where V is the velocity of substance concerned. The second term of the left of the equation (1) can be ignored, when the period of variation is large compared with the time taken by electromagnetic waves to travel across the region considered, and this is the case in our present problems, in which we concern only variations not very rapid.

In the dynamo-theory, the current system is obtained by solving the differential equation

$$\mathrm{rot}\,(i/\sigma - [V\,\mathrm{rot}\,A^{(E)}]) = 0, \tag{5}$$

where $A^{(E)}$ is the vector potential of the earth's permanent magnetic field. As to the air movement, it is assumed to be irrotational. When we concern somewhat rapid variations in § 5, the differential equation containing the term contributed from self-induction will be used instead of the above equation (5),

$$\mathrm{rot}\left(i/\sigma + \frac{\partial A}{\partial t} - [V\,\mathrm{rot}\,A^{(E)}]\right) = 0, \tag{6}$$

where the variational field itself is included in A in the second term.

In treating the electromagnetic induction in the ionosphere in § 4, will be solved the differential equation of the type

$$\nabla^2 A - \mu\sigma\frac{\partial A}{\partial t} = 0 \tag{7}$$

which can be deduced under some assumptions from the general equation (1).

§ 2. Longitudinal Inequality in Solar Diurnal Variation in Geomagnetic Field.[3]

In the analysis of the solar diurnal variation in geomagnetic field on quiet days, the assumption that it varies with local time, and not with longitude, has been adopted as a first approximation. Strictly speaking, however, the observed S_q-field differs considerably between stations in different longitudes. Therefore, for a given solar activity and season, S_q-field must be assumed to be a function of latitude, local time and longitude, or universal time instead of local time. A detailed picture of distribution of S_q-field over the earth at various Greenwich Time has been obtained by OTA[4] from the data of the Second Polar Year observations. According to their results S_q-field changes fairly regularly with the universal time.

The most conceivable cause of this longitudinal inequality in S_q-field seems to

be the effect of the discrepancy between the geomagnetic and geographic poles upon the dynamo-action in the ionosphere. Thus, the distribution of the current system was calculated by the dynamo-theory, the above-mentioned discrepancy of these two axis-poles being taken into account. In Fig. 1, four examples of calculated charts of equivalent electric current systems are shown. The intensity and position in latitude

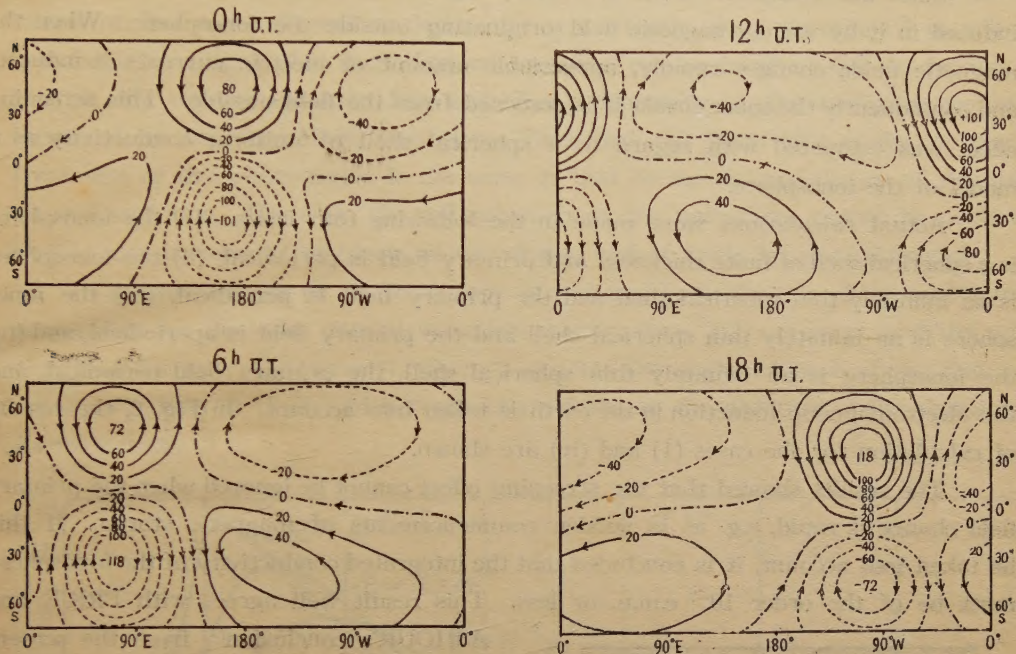

Fig. 1. Calculated charts of equivalent electric current-systems representing the external part of S_q at different universal times of sunspot-maximum year 1905. Current of 20000 amp flows between each two successive lines in direction indicated.

of the foci of equivalent current vortices obtained from the current system are in good agreement in their general tendency with the observed results. This argument can be considered as giving a proof of the generally accepted conception that the geomagnetic diurnal variation is caused by the dynamo-action in the upper atmosphere.

§ 3. Integrated Conductivity of the Ionosphere.

The integrated conductivity of the ionospheric region responsible for geomagnetic diurnal variation has been generally assumed to be of the order 10^{-5} or 10^{-6} e.m.u. These values, however, are too large compared with those deduced from the ionospheric observation, say 10^{-7} e.m.u. at the largest. Therefore, it is desirable, if possible, to estimate the integrated conductivity from other points of view. For this purpose, the conclusions from studies on the following two different problems will provide for the possibilities: they are the screening effect of the ionosphere especially in relation to magnetic storms and the solar flare type change in geomagnetic field. From the first it was concluded that the integrated conductivity of the ionosphere is of order of magnitude 10^{-7} e.m.u. or less, while, from the second that it is of the order 5×10^{-8}

e.m.u. The summaries of these discussions will be given in §§ 4 and 5. Thus we have to assume that the integrated conductivity is of the order $10^{-8} \sim 10^{-7}$ e.m.u., which is nearly equal to the value expected from the ionospheric observation.

§ 4. Screening Effect of the Ionosphere.[5][6]

Since the ionosphere is an electrically conducting shell, electric currents will be induced in it by varing magnetic field originating outside the ionosphere. When the magnetic fields changes rapidly, appreciable amount of electric currents is induced, and consequently the space inside it is screened from the field outside. This screening effect was estimated with regard to a spherical shell of uniform conductivity as a model of the ionosphere.

Actual calculations were made in the following four cases: (i) the ionosphere is a spherical shell of finite thickness and primary field is periodical, (ii) the ionosphere is an infinitely thin spherical shell and the primary field is periodical, (iii) the ionosphere is an infinitely thin spherical shell and the primary field is aperiodical, and (iv) the ionosphere is an infinitely thin spherical shell, the primary field periodical, and the electromagnetic induction in the earth is taken into account. In Fig. 2, the results of calculation for the cases (i) and (iv) are shown.

The results showed that the screening effect cannot be ignored when the primary field change is rapid, e.g. as in sudden commencements of magnetic storms. If this be taken into account, it is concluded that the integrated conductivity of the ionosphere must be of the order 10^{-7} e.m.u. or less. This result well agrees with PRICE and ASHOUR's conclusion[7] from the screening effect of the ionosphere of non-uniform conductivity.

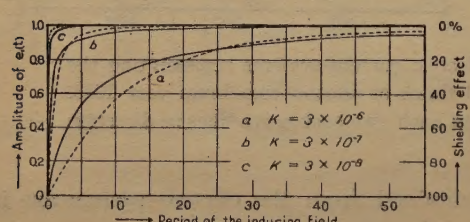

Fig. 2a. Screening effect of the ionosphere.
Ordinate: the amplitude ratio of the field inside the ionosphere to the primary field, when it is uniform, or the representation in terms of the screening effect in percent.
Abscissa: the period of the primary field.
K: the assumed integrated conductivity.
Full lines and broken lines correspond to the cases (iv) and (i) respectively.

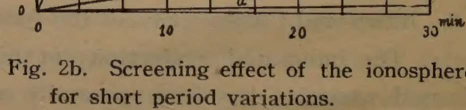

Fig. 2b. Screening effect of the ionosphere for short period variations.
Ordinate and Abscissa are the same as those in Fig. 2a.

The screening effect of the ionosphere in reference to other problems will also be of interest. The statistical result on the seasonal change in D_m by CYNK[8] may be explained qualitatively by the change in the ionospheric screening due to the seasonal change in conductivity. On the other hand, CHAPMAN and FERRARO[9] suggested in their paper on the radial stability of the hypothetical ring-current that the world-wide fluctuations of the storm-time field which occur during magnetic storms

might be produced by radial oscillations of the ring-current. If, however, the ionospheric screening is taken into consideration, this explanation seems to become rather untenable.

§ 5. Solar Flare Type Change in Geomagnetic Field.(10)

Since the solar flare type change in geomagnetic field is observed as an augmentation of the solar diurnal variation for a relatively short duration,(11) it will be reasonable to assume that such changes are given rise to by the same mechanism as that of the diurnal variation but with the conductivity of the ionoshere suddenly increased by the anomalous ionization. Therefore, the basic idea of the theoretical treatment of this phenomenon is the same as that of the dynamo-theory of the solar diurnal variation. However, the self-induction of the ionosphere cannot be neglected in this case because of rapidness of the change and of fairly large conductivity.

The conductivity of the ionosphere is assumed to be the sum of the conductivity responsible for the solar diurnal variation in quiet days, $\rho_0(1+\varepsilon_0 \cos \chi)$, and the additional conductivity, $\Delta\rho(1+\cos \chi)$, where χ denotes the zenith distance of the Sun and $\Delta\rho$ is a function of time, which is directly related to the duration of the change and its magnitude.

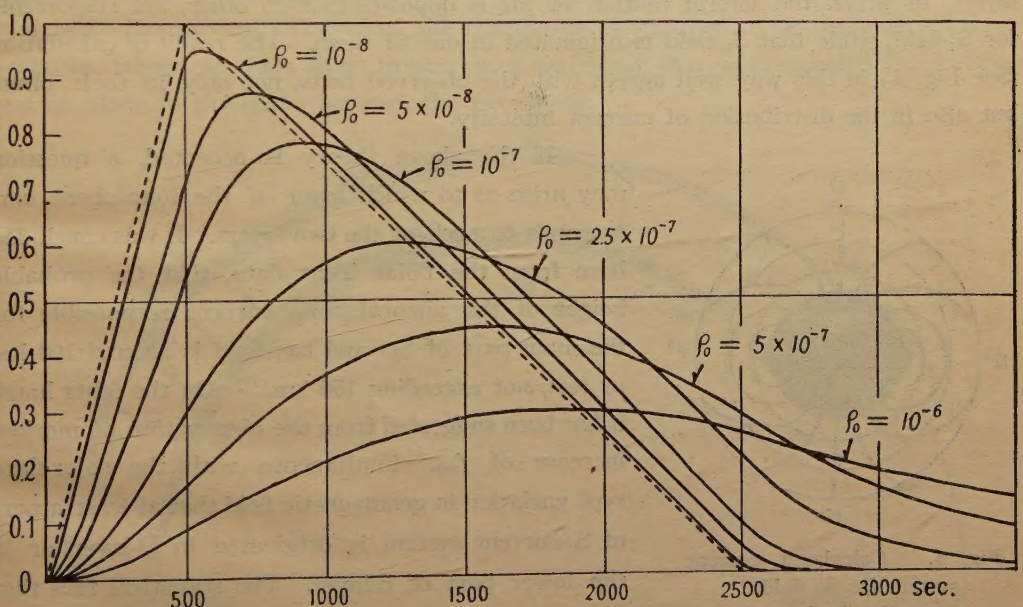

Fig. 3. Solar flare type change. Ordinate is the rate of augmentation of the coefficient of the current function for various assumed integrated conductivities. Abscissa is time in second. The change $\Delta\rho/\rho_0$ assumed is shown by broken line.

The problem is, then, reduced to extending the dynamo-theory, by which transient phenomena can be dealt with comprehensively. This was performed by the help of perturbation method.

The magnitude of the change and the time required by it to reach its maximum, when compared with the observed data, give the range of the admissible value for

conductivity (See Fig. 3.). The mean integrated conductivity thus determined is 5×10^{-8} e.m.u., which is consistent with the result from the screening effect of the ionosphere.

§6. The Current System S_D- and Bay-fields.

The S_D-current system can be obtained theoretically together with the S_q-current system under the assumption that the auroral zones are especially of a high conductivity compared with the lower latitude region and with the polar cap. The result of calculation is in a fairly good agreement with the observed one, especially in their general mode.(12) The bay-field can also be interpreted almost in the same way, except that the duration of the induced E.M.F. in the auroral zones is limited within a few hours.

It was found, however, that the calculated current-system for S_D-field differs definitely from the observed one in their phase; that is, the theoretical current system, though the S_q-part of which just agrees with the observed S_q-field, differ from the Chapman-Vestine's system(13) by about 6 hours in phase, and by 10 hours from the one recently obtained by us(14) as well as from the Vestine's bay-system.(15)

One of the ways to avoid the difficulty mentioned above is to assume that two layers, in which the lateral motion of air is opposite to each other, are responsible for S_q-field, while that S_D-field is originated in one of them. The result of calculation (See Fig. 4.) in this way well agrees with the observed facts, not only in their phase but also in the distribution of current intensity.

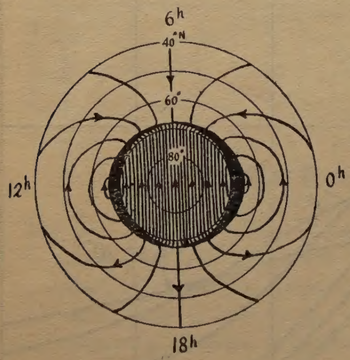

Fig. 4. Calculated current system for S_D field.
The distribution of conductivity and the air motion in the conductive layer are assumed appropriately so as to make the calculated current system fit the observed one. Electric current of 1.17×10^4 amp. flows between successive stream lines in direction indicated.

If the above theory is accepted, a question may arise as to which layer of the ionosphere corresponds to each of the two layers. It was concluded here from the Polar Year data, that the probable height of the auroral zone current responsible for the main part of S_D- and bay-field is around 100 km or less, not exceeding 150 km.(16) On the other hand, it has been suggested from the observation of marked increase of f_{min} simultaneous with the solar-flare type variation in geomagnetic field that at least a part of S_q-current system is originated in D-layer or in the lower part of E-layer. The statistical fact that the range of S_q-variation is proportional to $(f_{min})^2$ shows also that the main part of S_q-current is flowing in the above-mentioned lower layer.

Summarizing these results, we may presume at present that S_D-current system is situated in the upper part of E-layer, while S_q-field is the resultant field due to the current systems both in D- and E-layers.

§ 7. Dynamic Motion of the Ionosphere Region.

The conclusion obtained above leads us to assume a large velocity of the dynamical motion of the ionosphere, so far as we take the dynamo-action in the ionosphere as the main cause of S_q-, S_D-, L-, bay- and solar-flare variations in geomagnetic field. That is to say, if we take 5×10^{-8} e.m.u., in place of 3×10^{-6} e.m.u. by Chapman and Schuster, as the average integrated conductivity of the ionosphere concerning geomagnetic variation, the lateral velocity of the ionosphere motion should be of order of magnitude 30 m/sec. If further we assume the double layer theory mentioned in § 6, the integrated conductivity of one of the two layers must be 20~30% larger than that of the other, and the lateral velocity of each layer must amount to 100~150 m/sec, or 360~540 km/hour.

On the other hand, that E- and F-layers are moving laterally with velocity of 100~600 km/hour has recently been reported by various authors, including one of the present writers.[17] And this result seems to agree with our theoretical expectation.

Here, it may be presumed that the said dynamical motion of the ionosphere region will be rather a general circulation, and not merely the tidal current, since the amount of velocity is too large to be the latter. In Fig. 5, an example of the circulation of ionospheric air, which is in the way of dynamo-theory responsible for S_q-field, is shown. In practical calculation, the circulation is assumed to be composed of three parts, i.e. lateral motions alone in the outer and inner thin spherical shells and radial motion alone in the space between the two shells.

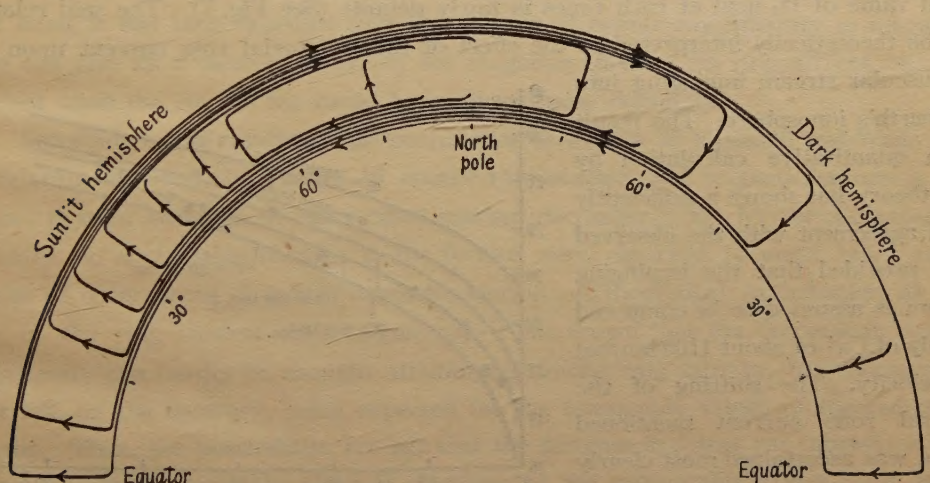

Fig. 5. Calculated circular air motion of the upper conductive layers in the northern hemisphere of 169°E—11°W meridian. The stream lines whose ends vanish in the figure mean those closed in the space in both sides of the considered meridian.

§ 8. Development of S_D- and Bay-fields

The studies of progressive changes in S_D- and bay-fields will be significant for getting the dynamical view of these phenomena. The result of statistical examination of the successive aspects in the whole course of the average bay in the middle latitude

region has shown characteristics of its progressive change. The bay current system in the dark-hemisphere generally grows with time from the beginning, while the current system in the sunlit-hemisphere, on the contrary, is contracted (See Fig. 6), resulting in the well-known looping phenomenon of the horizontal vector-arrow of the bay-disturbance. This phenomenon is, theoretically speaking, due to the development of the auroral zone current in the dark side.

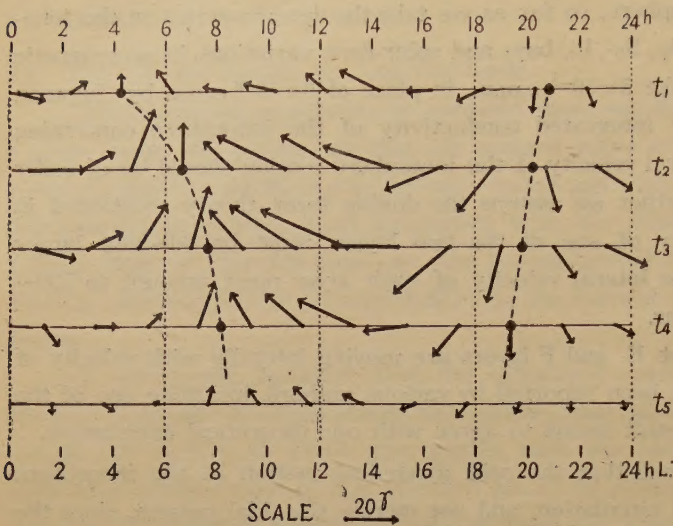

Fig. 6. Progressive change in the distribution of current arrows of the mean bay disturbance at the latitude of Toyohara (46°58′N, 142°45′E). The times of its beginning, middle and end are denoted by t_0, t_3 and t_6 respectively. Full circles in the figure correspond to the boundaries of the positive and negative bay sides.

On the other hand, hourly aspect of development of S_D-field was examined especially in detail with the Polar Year data.[18] The relation between the polar (magnetic) distance of the auroral zone and the equatorial value of D_{st}-field at each times is fairly definite (See Fig. 7). The said relation can be theoretically interpreted as the effect of the equatorial ring current upon the corpuscular stream impinging into the earth's ionosphere. The result of a quantitative calculation by this theory has shown a sufficiently good agreement with the observed fact, provided that the impinging stream is assumed to be composed mainly of Ca^+ of about 1100 km/sec in velocity. The shifting of the auroral zone current mentioned above was ascertained most clearly and in detail as to the magnetic storm of May 1, 1933, and the same character was found in the other storms in the Polar Year as well as in those in 1949.

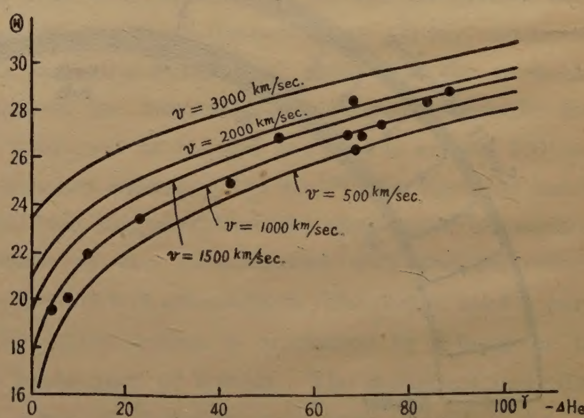

Fig. 7. Relation between the polar distance of the auroral zone in angle of latitude (Θ) and the decrease in the equatorial horizontal intensity (−ΔHe), which is assumed to be caused by the growth of ring-current at the distance of 1 Störmer's unit.

The curves show the calculated results for Ca^+ as an impinging particle of various velocities. Full circles are the observed values during the magnetic storm of Apr. 30–May 1, 1933.

One of interesting facts obtained is that the S_D-current system seems to begin to take place soon

after the sudden commencement of magnetic storms, though the intensity in the said early stage is not so intense as in the main part of the storm. This conclusion was obtained from the world wide data of hourly values of magnetic storms in the Polar Year and from the magnetograms of recent storms collected by the Ionosphere Research Committee. Then, we may say that Chapman-Ferraro's theory of the initial part of magnetic storm will be under the necessity of re-examination, when the above-mentioned fact as well as the screening effect of the ionosphere was taken into consideration.

§ 9. Some Remarks on Magnetic Disturbances.

As to the disturbances of F_2-layer, the result of our studies on Japanese data[19] has shown that the ionosphere storms in F_2-layer generally follow magnetic storms with some delay, the former's characteristics being subjected also to local time as well as to season. This result may lead us to presume that the ionosphere storm in F_2-layer is a secondary phenomenon accompanying magnetic storm, and does not or very little contribute directly to the electric current producing the disturbances in geomagnetic field. Thus, it seems that the lower parts of the ionosphere are chiefly responsible for various geomagnetic variations.

One of the significant problems concerned here will be the character of the ionizer of the ionosphere, especially the corpuscular stream from the Sun. The dynamical behaviour of the auroral zone and its relation to D_{st}-field, such as dealt with in § 8, seem to provide us with a suggestion about the nature of the corpuscular stream, especially about the effect of the supposed equatorial current ring. On the other hand, it has been expected that the relation of cosmic ray intensity to magnetic storm can give us a clearer picture of the magnetic effect of the ring current, because its effect upon the cosmic ray could be analyzed more definitely.

According to the results of our examination as well as of Miyazaki and Wada's,[20] the relation between the change in cosmic ray intensity and magnetic storms is not unique; that is, although in many cases marked decreases in the former accompanied the latter, no appreciable changes in the former were observed in many other cases in spite of occurrences of severe magnetic storms. A theoretical examination of the effect of the ring current upon cosmic ray[21] has shown that the decrease in cosmic ray intensity can hardly be thought attributable to the ring current, the increase of about 2%, on the contrary, being expected for the reasonable values of the radius of the ring. Then, the possibilities are left that the decrease in cosmic ray intensity might suggest something about the physical character of the corpuscular stream itself apart from the magnetic effect of the ring current formed by it.

§ 10. Conclusion.

The present writers are believing now that the dynamo-action is the most fundamental process to produce various electric current systems in the ionosphere, which are subjected to various given conditions of ionization there. And the geomagnetic disturbances observed on the earth's surface must also depend much upon the ionospheric condition, as was dealt with in §§ 4 and 6.

It seems, however, that there remain various difficult problems especially concerning geomagnetic disturbances, which should be our main items of future researches.

In conclusion, the writers wish to express their sincere thanks to Miss C. Aikawa and Miss N. Ono for their assistance in numerical computations in the studies of §§ 2, 5, 7 and 8, and also to Mr. T. Suzuki and Mr. S. Nagumo for their assistance in calculation of a part of § 7.

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On the Electrical Conductivity of the Upper Atmosphere

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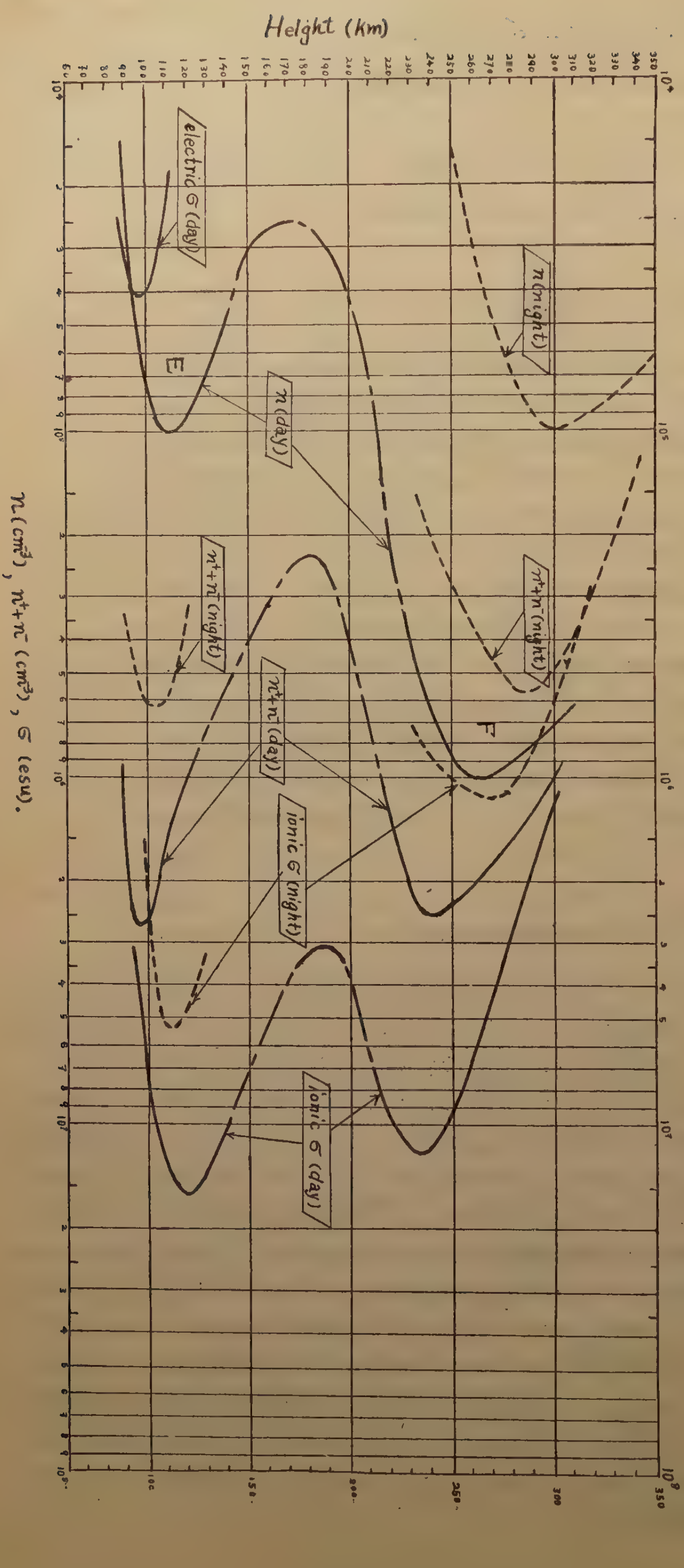
Abstract

The paper deals with the problem of the electrical condnctivity supported by the upper atmospheric ionization, to which the diurnal variation of terrestrial magnetism is attributed in the dynamo theory. Number densities of free electrons and ions and their collision frequencies are substantially important for the evaluation of the vertical distribution and integrated value of the electrical conductivity. For this purpose various elementary processes governing the ionospheric characteristics, that is, recombination, attachment, detachment and collision, are discussed on the basis of ionospheric observation with the aid of some theoretical data by other scientists. Although our present knowledges had to face some contradictions on the way of numerical estimation of necessary elements, it is indicated that the total conductivity will be about 2×10^{-7} e.m.u. during the daytime, with the maximum conductivity of $(1.5\sim2.0)\times10^{-14}$ e.m.u. in the E-region.

I. Introduction

Although many important points on the physical relations between the ionosphere and the terrestrial magnetism have been found up to this time, there still remains the problem of the electrical conductivity of the ionosphere, which has a significant bearing to the physical nature of diurnal variation of the terrestrial magnetism. From the analysis of diurnal variations observed in the terrestrial magnetism it was shown that the total conductivity of the upper atmosphere should be of the order of 10^{-6} e.m.u. Recently, however, T. Nagata and others[1] reported the results of study on ionospheric induction effect, and according to their conclusions it will be reasonable to accept that the total conductivity will be less than 10^{-7} e.m.u. Sir Edward Appleton[2] gave a formula with a table to evaluate the total conductivity based on the recombination theory and we can calculate the total conductivity by assuming collision frequencies, number densities of electrons and ions and scale height. As to the evaluation of density ratio of electron to ion, which is very important to our present problem, we can refer to the reports by G. Goubau,[3] F.L. Mohler,[4] D.R. Bates and H.S.W. Massey[5] and others.



The writer discusses here some problems necessary to find out the density ratio of electron to ion and collision frequencies on the basis of ionospheric measurements together with some theories of other scientists, and the vertical distribution and the integrated value of the electrical conductivity and their changes from day to night are presented.

II. Mean collision frequencies

1. Theoretical valves

By theoretical calculaion of T. Yamanouchi[(6)] the mean collision frequency (ν) of free electron in F_2-layer, where Oxygen atom is the controling particle, is given as

$$\nu \simeq 3\times10^{2}\ \mathrm{sec}^{-1}\quad (\mathrm{h}=250\ \mathrm{km}).$$

In this case the number density of Oxygen atoms is assumed to be about 6×10^{9}/c.c. and the temperature as about 1000°K.

2. Observed values

According to the experiments by Farmer, Ratcliffe and others[(7)] we have the values for electron collision as follows.

$$\nu \simeq 10^{5}\ \mathrm{sec}^{-1}\quad \text{for } E\text{-layer (100 km)},$$

$$\nu \simeq 1.5\times10^{3}\ \mathrm{sec}^{-1}\quad \text{for } F_2\text{-layer (250 km)}.$$

As to the F_2-layer we consider the contribution of electronion collision according to T. Yonezawa[8] and adopt the value of ν a little less than the above value, that is, $(3\sim10)\times10^2$ for F_2-region.

3. Height dependence

Height dependence of ν for electron collision is given by the following relation,[6]

$$\nu \backsim N\sqrt{T}\,, \tag{1}$$

where N is the number density of neutral particles and T the electron temperature. For the purpose we have to know the vertical distributions of N and T, but it is very difficult at present stage to show them exactly. The following table is presented to show the values of ν for each layer, together with other various values, with which we consider the problems as our model ionosphere.

Table of Model Ionosphere

Region	Height (km)	Gas	Temperature (deg. Kelvin)	Pressure (mmHg)	Gas density N (cm^{-3})	Mean collision frequency $\nu\ (sec^{-1})$	Attachment coefficient $\beta\ (sec^{-1})$	Apparent recombination coefficient $\alpha'\ (cm^{-3}sec^{-1})$
E	90	O_2 & O	400	2×10^{-3}	1.4×10^{14}	4×10^5	7×10^{-2}	10^{-8}
	100			5×10^{-4}	3×10^{13}	1×10^5	2×10^{-2}	
	110			2×10^{-4}	1×10^{13}	4×10^4	8×10^{-3}	
	120			1×10^{-4}	5×10^{12}	2.3×10^4	5×10^{-3}	
	200	O	800	2.8×10^{-6} (1.6×10^{-5})*	4×10^{10} (2×10^{11})**	3×10^3	5×10^{-4}	10^{-9}
	220			1.5×10^{-6} (1.0×10^{-5})*	2×20^{10} (1.3×10^{11})**	2×10^3	3×10^{-4}	
	250		1.000	6×10^{-7} $(5.0\times10^{-6}$	7×10^9 (6×10^{10})**	1×10^3	1.4×10^{-4}	10^{-10}
	400			2×10^{-7} (1.5×10^{-6})*	2.2×10^9 (2×10^{10})**	3×10^2	4×10^{-5}	

( )* Values by P.O. Pedersen.[9]
( )** Values corresponding to the above.

4. Collision of ion with neutral particles

As we have no exact ground for the estimation of mean collision frequency of ion with neutral particles, except that we know the ratio of mean collision frequencies of electron and ion at the normal pressure, we assumed that the above ratio may hold up to the height of F_2-layer. The ratio is taken as about 40.[9]

III. Ionic density

1. Mathematical relations of electronic and ionic densities

As is easily shown, the following relations governing the densities of electron, negative and positive ions are given, taking attachment and detachment as well as recombination processes into account.

$$\frac{dn}{dt}=q-\alpha n^{+}n-\beta n+\gamma n^{-}+\delta n^{-}, \tag{2}$$

$$\frac{dn^-}{dt} = \beta n - \gamma n^- - \delta n^- - \alpha_i n^+ n^-, \tag{3}$$

$$n^+ = n^- + n, \tag{4}$$

where

n, n^-, n^+....number densities (cm^{-3}) of electron, negative and positive ions respectively,

q ....ion production rate (cm^{-3}. sec^{-1}) by solar radiation,

α ....recombination coefficient (cm^3. sec^{-1}) of electron and positive ion.

α_i ....recombination coefficient (cm^3. sec^{-1}) of negative and positive ions,

β ....attachment coefficient (sec^{-1}) of electron to neutral particle,

γ ....photo-detachment coefficient (sec^{-1}) of electron from negative ion,

δ ....detachment coefficient (sec^{-1}) of electron from negative ion by collision.

During the night we may put $q = \gamma = 0$.

2. Attachment coefficient (β)

Concerning β for F-region, some theoretical results[6][10] on attachment cross section of Oxygen atom and many experimental data[11] are available, but as for E-region, the value is speculative. Considering simply the pressure dependence of β we use the values in the above table for our present work.

3. Recombination coefficient (α and α_i)

Besides the theoretical formulas, a great many experimental data[12] are available. Considering that the observed value of α is actually the apparent value α', which is equal to $(1+\lambda)\alpha$ ($\lambda = n^-/n$), we must be carefull to utilize the experimental values.

From experiments we may take α' as follows.

$$\alpha' \simeq 1 \times 10^{-8} \ cm^3 \, sec^{-1} \quad \text{for } E\text{-region},$$

$$\alpha' \simeq 1 \times 10^{-10} \ cm^3 \, sec^{-1} \quad \text{for } F\text{-region}.$$

Concerning α_i we have little knowledges,[9] and in the following considerations we touch this quantity only little, though in the more detailed study α_i should be taken into account.

4. Criterion from the increase of electron density of F_2-layer near the sunrise at ground.

When we consider the rapid increase of F_2-layer electron density from the sunrise at F-layer height (Ca. 250~300 km) to near the one at ground, which is very remarkable especially in Winter (from November to February), we can explore rough values of $\gamma+\delta$ (detachment) and then n^+ and n^-.

As q is zero for the above period of day, we obtain the following equations from equation (2).

$$\frac{dn}{dt} = (\gamma+\delta)n^- - an^+n - \beta n, \tag{5}$$

$$\simeq (\gamma+\delta)n^+ - (\gamma+\delta+\beta)n, \tag{5'}$$

where an^+n is neglected in comparison to βn.

If we consider n^+ as constant for the period under consideration, the above equation is solved as follows.

$$n \simeq \frac{\gamma+\delta}{\beta+\gamma+\delta} n^+\{1-e^{-(\beta+\gamma+\delta)t}\}+n_0 e^{-(\beta+\gamma+\delta)t}, \qquad (6)$$

where n_0 is the value of n at the sunrise of F-region height.
The above equation is rewritten in the following form.

$$n/n_0 \simeq \frac{\gamma+\delta}{\beta+\gamma+\delta}(1+\lambda)\{1-e^{-(\beta+\gamma+\delta)t}\}+e^{-(\beta+\gamma+\delta)\lambda}. \qquad (7)$$

When we use the value 10^{-4} as β, and 3×10^{-3} as γ, which is reported by some theoretists,[9] (δ is taken as less than β), we get for n/n_0 after 333 seconds (about 5.5 min.),

$$n/n_0 \simeq 0.63(1+\lambda)+0.37.$$

This value is too much, unless λ is taken as very small compared with unity. But with such small λ the actual observed curve of n/n_0 deviates greatly from the equation (7) for the later part of the period. Then it may be supposed that γ (including δ) should be as low as of the order of 10^{-4}, but in this case the constancy of λ or n^+ is violated.

In treating the equation (5′), we may proceed on another way as follows. The equation (5′) can be written in the following form,

$$\frac{dn}{dt}=(\gamma+\delta)\lambda n-\beta n. \qquad (5'')$$

Considering λ as variable with time we get the solution,

$$\frac{n}{n_0}=e^{(\gamma+\delta)\int\lambda dt-\beta t}, \qquad (8)$$

and for n^- the solution (n_0^- is the initial value of n^-),

$$\frac{n^-}{n_0^-}=e^{\beta\int\frac{1}{\lambda}dt-(\gamma+\delta)t}. \qquad (8')$$

When we compare the above equation (5″) or (8) with the actual increase of n in F_2-layer in Winter by taking β as 10^{-4}, we find that the value of $\gamma+\delta$ as the order of 10^{-4} with λ of the order of unity will be the most probable, but $n^+(=n+n^-)$ varies considerably in this case and this is unreasonable for the period under consideration.

Then it may need, so far as the coincidence of the theoretical equations and the actual observed curves is concerned, that both $\gamma+\delta$ and λ are considered to vary for the period, subject to constancy on n^+. In any case the coincidence is more or less insufficient, but it is recommendable that γ (including δ) should be taken as $10^{-4}\sim3\times10^{-4}$ with $\lambda\simeq1\sim3$.

And in these estimations we find also that λ just before the sunrise at F_2-layer height will be of the order of $3\sim10$, which may ae the night value for λ.

In these considerations the heights of maximum electron density of F_2-layer for day and night are taken as 260 and 300 km respectively and the values of electron density at constant height are used for the calculation.

5. Ionic density of F_2-region

In the daytime, we get the following relation for around noon, which the

electronic density rearches near its maximum value.

$$n=\frac{q+(\gamma+\delta)n^-}{\beta+\alpha n^+}. \qquad (9)$$

And as a prelimimary approximation we have the relation,

$$n\simeq\frac{\gamma+\delta}{\beta+\gamma+\delta}n^+. \qquad (9')$$

Using the values

$$\beta=1.0\times10^{-4} \quad \text{(for 260 km)},$$
$$= 4\times10^{-5} \quad \text{(for 300 km)},$$
$$\gamma+\delta=10^{-4}\sim3\cdot10^{-4},$$

we get

$$n\simeq\frac{1}{2}n^+\sim\frac{3}{4}n^+ \quad \text{(for 260 km)},$$
$$\simeq\frac{2}{3}n^+\sim n^+ \quad \text{(for 300 km)},$$

and then

$$n^++n^-\simeq 3n\sim\frac{5}{3}n \quad \text{(for 260 km)},$$
$$=2n\sim n \quad \text{(for 300 km)}.$$

For the above values the equation (9) holds roughly with the magnitude of $q\simeq100$.

In the night, when the electronic density decreases to its final value, we get with neglection of αn^+n,

$$\lambda=\frac{n^-}{n}=\frac{\beta}{\delta}. \qquad (10)$$

Although δ is not exactly known, δ must be less than 10^{-4} by the consideration of the preceding paragraph, and we assume n^-/n as $2\sim10$ (the former for 300 km and the latter for lower than 250 km.) and consequently n^++n^- as about $5\sim20$ times of n during night for F-layer.

6. Ionic density of E-region

For E-region β was given in the table. In the daytime we have from various experiments the following relation of electron density (n) around noon, ($\frac{dn}{dt}$ is put to zero)

$$q\simeq\alpha' n^2=(1+\lambda)\alpha n^2. \qquad (11)$$

As the above relation is fairly accurate, the following relation must hold from the equations (2) and (11).

$$\beta n\simeq(\gamma+\delta)n^-,$$
$$\therefore\ \frac{n^-}{n}=\frac{\beta}{\gamma+\delta}. \qquad (12)$$

If we take γ as independent of height ($10^{-4}\sim3\times10^{-4}$) and δ as 10^{-3}, we get following results.

$$\lambda=\frac{n^-}{n}\simeq20\sim15 \quad \text{(for 100 km)},$$
$$\simeq 7\sim6 \quad \text{(for 110 km)}.$$

And for the night, if we neglect δ, we get the following equation from (3), considering $\frac{dn^-}{dt}=\gamma=0$,

$$n/n^+n^- \simeq \frac{\alpha_i}{\beta}, \tag{13}$$

and then

$$\lambda = \frac{1}{\sqrt{n}}\sqrt{\frac{\beta}{\alpha_i}}. \tag{14}$$

Putting $\alpha_i \simeq 10^{-9}$ and n as less than 10^{-1}, we get

$$\lambda > 45 \quad (\text{for } 100 \text{ km}),$$
$$\lambda > 28 \quad (\text{for } 110 \text{ km}).$$

And when only δ is taken into account (α_i; neglected), λ will decrease from the above values to about 20 and 8 respectively, and therefore the above values of λ will be the upper limits.

As the consequence we have

$$\left.\begin{aligned} n^+ + n^- &\simeq 15\sim 13,\ n \text{ (average 14) (for 110 km)}, \\ &\simeq 41\sim 31,\ n \text{ (average 36) (for 100 km)}, \end{aligned}\right\} \text{for daytime,}$$
$$\left.\begin{aligned} &\simeq 60\ n \text{ at most (average 30) (for 110 km)}, \\ &\simeq 90\ n \text{ at most (average 60) (for 100 km)}. \end{aligned}\right\} \text{for night.}$$

IV. Vertical distribution curves

1. n and $n^+ + n^-$

From the above considerations we reached the stage to draw vertical distribution curves of necessary values. For the distribution of electron density n (lower than the maximum density) we used the observational data and for the other part the square root curve of ion production q by Chapman's theory was used. In the figure shown the maximum electron densities of E and F_2-layers for daytime are taken as 10^5 at 110 km and 10^6 at 260 km respectively. From this curve we draw the curve of $n^+ + n^-$, where the height dependency of necessary elements is taken into account, and the curve is shown in the figure.

2. Electrical conductivity (σ)

The electrical conductivity σ of the ionosphere is given by the following equation.

$$\sigma = \frac{e^2 n}{m}\frac{\nu}{\omega_H^2+\nu^2} + \frac{e^2(n^+ + n^-)}{M}\frac{\nu_i}{\omega_{IH}^2+\nu_i^2} \text{ esu}, \tag{15}$$

where

ν_i......mean collision frequency of ion with neutral particle,
ω_H......geomagnetic gyrofrequency for electron ($\sec^{-1}$),
ω_{IH}......geomagnetic gyrofrequency for ion ($\sec^{-1}$),
m, M......mass of electron and ion respectively,
e......charge of electron.

In the calculation of σ we used the following values.

$$M/m = 3\times 10^4,$$
$$\omega_H = 6.6\times 10^6 \quad (H = 0.4 \text{ Gauss}),$$
$$\omega_{HI} = 2.3\times 10^2 \quad (\quad ,, \quad\quad ,, \quad).$$

The other values necessary for the calculation of σ can be seen in the figure and the table. The vertical distribution of σ for daytime is calculated and shown in the same figure. As the electronic conductivity is very small in both layers, only E-layer daytime conductivity of electron is shown.

According to these curves and some numerical calculations based on the curves the followings can be said. For the daytime the maximum conductivities of E and F-layers are 1.7×10^{-14} e.m.u. and 1.3×10^{-14} e.m.u. respectively and the integrated values for E and F-layers are 1.1×10^{-7} e.m.u. (60%) and 0.7×10^{-7} e.m.u. (40%) respectively, the boundary between the two layers being taken as 190 km. The total is abou 1.8×10^{-7} e.m.u. The conductivity is rather sensible for the height, and the decrease of F-layer height and the increase of E-layer height make the conductivity increase.

For the night the total conductivity of E-layer decreases roughly to less than one fifth of the daytime value and that of F-layer to one eighth, because the increase of height at night decrease σ greatly. The over all conductivity is about 3.15×10^{-8} e.m.u. at most, i.e. one sixth of the daytime value. The situation (mayimum condition) during night is shown by the dotted curves in the same figure. (The explanation is omitted.)

V. Concluding remarks

It is to be noticed that the conductivity during daytime depends upon the ionic density of E and F-layers instead of electron density. The contribution of E-layer is greater than that of F-layer. During night the contribution of E-layer is much greater than that of F-layer. The diurual change of the conductivity is represented partly by the electron density variation of E-layer, to which the ionic density of E-layer is closely connected, and partly by the height and electron density variation of F-layer.

Although it must be said that some dangerous speculations are involved in this estimation of the electrical conductivity, the study will show an example of the efforts that can be done if one aims to estimate the electrical conductivity directly from the knowledge of ionosphere. We expect further studies on this line and the consideration of calm and disturbed time conditions of ionosphere and terrestrial magnetism.

In conclusion the writer wishes to express his thanks to Dr. T. Nagata and Mr. Yonezawa for their valuable advices.

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Diurnal and Seasonal Frequencies of Occurrence of 'Sudden Commencements', SC, in Geomagnetism

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Abstract

By the statistical investigations of the diurnal and seasonal frequencies of the occurrence of ordinary and second-type SC's at Kakioka in the period 1926–1949, the auther found the following characteristics. Only in the case of larger amplitude of SC, $\varDelta H \geqq 30^{\gamma}$, he found the similar type of diurnal variation with that obtained by Ferraro and Parkinson at stations in middle latitudes, $\varDelta H$ being meant by the first impulsive change of SC in the horizontal force. In the case of smaller amplitudes, however, matters are not so simple that, for instance, he found an inverted type in the case of small SC, $\varDelta H \leqq 10^{\gamma}$. As a whole, however, the number of occurrence of SC in daytime increases with decreasing $\varDelta H$ relative to that in night, which may be due to the shielding effect of the ionosphere for the external field.

The number of occurrence of the second-type was very small and nine percent when only sharp preliminary impulses were counted. In Ferraro-Parkinson's figure 2, Greenwitch, Watheroo and Kakioka lie nearly on a smooth curve.

Regarding to the seasonal variaton, the auther obtained two types, one of which is an ordinary double oscillation type with two maxima in equinoxes, while the other in summer and winter, corresponding to the maximum and minimum periods of the sunspot activity, respectively.

1. Introduction.

Recently, H. W. Newton[1] and V. C. Ferraro and W. C. Parkinson[2] reported an interesting and important fact that the mean diurnal frequency of occurrence of SC has a broad minimum around 8 hr. in local time, but not in universal one, which seems to be contrary to the usual conception of a world-wide phenomena of SC. G. Ishikawa[3] studied a similar diurnal variation of the amplitude of SC in the horizontal intensity from the standpoint of the electromagnetic shielding effect of ionospheres. Ferraro and Parkinson also pointed out that at the stations in low latitudes the diurnal variation of frequency such as appeared at Greenwitch is apparent only for SC's which

are followed by large or moderate disturbances. The available data at Kakioka, geomagnetic latitude 26°.0 N, longitude 154°.0 W, support this last view. The auther, therefore, tried to make more precise statistical investigations of the characteristics of the frequency of SC in low latitudes. Data used here was obtained at the Kakioka Magnetic Observatory in the period 1926–1949.

2. *Frequency of occurrence of SC classified in its amplitude, sunspot activity and magnitude of disturbance accompanied with it.*

At first, from the adopted three hundred and ninety two SC's of both ordinary and second-type of increasing horizontal intensity are arranged the mean numbers of occurrence per year in the following table 1. Although the above number might be increased by a few percent by taking such ones into counting that were omitted for the reasons of their small or irregular amplitudes, their occurrences in the course of proceeding storms, and other reasons, the generality of the treatment would not be lost.

Table 1. Mean number of SC per year.

Sunspot	$\Delta H \geqq 30^\gamma$	$10^\gamma < \Delta H < 30^\gamma$	$\Delta H \leqq 10^\gamma$
Total max. period	4.7	12.0	3.6
Total min. period	0.6	6.9	3.8

The amplitude of SC in the horizontal intensity, $\varDelta H$, is classified into three groups, and for the maximum periods of the sunspot activity are adopted the following three intervals, 1926–1930, 1937–1941 and 1946–1949, and corresponding minimum periods consist of two intervals, 1931–1937 and 1942–1945. It is noticeable from the table that number of SC is largest in the group $10^\gamma < \varDelta H < 30^\gamma$, and larger SC's are more frequently occurred in the maximum periods than minimum ones, while smaller SC's appear to be almost independent of the sunspot activity.

For the second, the number of occurrence classified for the magnitude of disturbance accompanied with SC is shown in the table 2. The magnitude of each disturbance is expressed by the daily maximum range of the horizontal intensity in G.M.T. on the very day of SC, or next day, of which larger value is adopted. As a whole, the table shows that larger SC's are more frequently followed by larger disturbances than smaller SC's.

Table 2. Number of SC for the magnitude of disturbance.

SC	Magnitude of disturbance	
	$>100^\gamma$	$<100^\gamma$
$\Delta H \geqq 30^\gamma$	52	19
$10 < \Delta H < 30$	74	160
$\Delta H \leqq 10$	23	64

Similar treatment in respect to the duration of the impulsive change as well as the amplitude may be needed for the further investigations of SC, but here are all neglected. At any rate, these general characteristics said above should be taken into considerations of the statistical investigation of the frequency of occurrence and also amplitude of SC.

3. *Diurnal variation of frequency of occurrence of SC.*

The diurnal curves of SC's were obtained by counting the number of SC occured

in each three hours' interval in respect to different groups of ΔH and different periods of sunspot activity. Fig. 1, or table 3 shows such a diurnal variation of SC in the case

Table 3. Diurnal variation of occurrence of SC, $\Delta H \geqq 30^{\gamma}$.

135°E Meridian Mean Hour

Hour	0	3	6	9	12	15	18	21
Percent	23.1	13.8	4.6	4.6	10.8	13.8	9.2	20.0

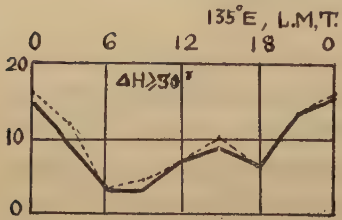

Fig. 1. Number of occurrence.
...... Total period.
—— Sunspot max.

of $\Delta H \geqq 30^{\gamma}$. It has a broad minimum in the morning and smaller one in the evening.

This mode of diurnal variation is similar with that obtained by Ferraro and Parkinson at stations in middle latitudes, if we construct the three hours' curve from their hourly value's one, say curve I.[2] This type is not only appeared in this total period, but also similar variation is noticeable in each maximum period mentioned above, if we permit less smoothness of each curve due to the smaller number of data. These variations are shown in the following table 4 in respect to the percent of occurrence of SC in day and night. We have a significant difference of percents between day and night. We should like to call hereafter this type as 'N-type' variation.

Table 4.

Hour	Period		
	1926–30	1937–41	1946–49
0, 3, 18, 21	33.3	32.0	38.3
6, 9, 12, 15	66.7	68.0	61.7

For the small SC, $\Delta H \leqq 10^{\gamma}$, the diurnal variation in the total period is drawn in Fig. 2 or table 5, in which refered curve F-P is the three hours' frequency reconstructed by the auther from the original curve I of Ferraro-Parkinson's paper.[2] For simplicity, we call this inverted type as 'I-type' variation. The correlation coefficient r between the N-type and I-type is $r = -0.84$;

Table 5. Diurnal variation of occurrence of SC, $H \leqq 10^{\gamma}$.

Hour	0	3	6	9	12	15	18	21
Percent	3.4	9.2	13.8	19.5	12.6	10.7	21.9	9.2

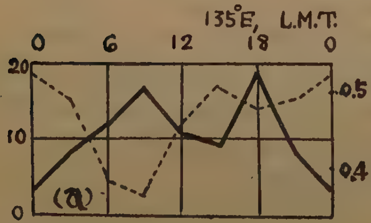

Fig. 2a. Number of occurrence.
—— $\Delta H \leqq 10^{\gamma}$.
...... F-P.

that is, the probability Pr such that the correlation coefficient of their populations ρ lies in the interval, $0.34 \leqq \rho \leqq 0.96$, is $Pr = 0.95$, while the similar coefficient between I-type and F-P curve is $r = -0.65$. Fig. 2(b) shows the similar variations respectively for the total sunspot maximum and minimum periods. Their percents of the numbers of occurrence corresponding to the following two groups of hours are as follows,

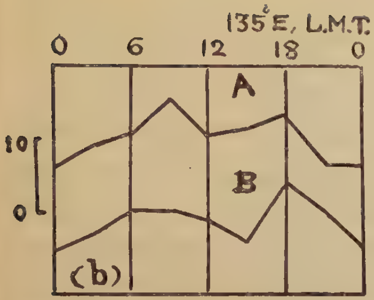

Fig. 2b. Number of occurrence.
A : Sunspot max.
B : Sunspot min.

Hour	max. period	min. period
0, 3, 15, 21	33.3	30.8
6, 9, 12, 18	66.7	69.2

Thus from the statistical point of views, the I-type variation is reliable.

The diurnal variation of frequency of occurrence of SC for the intermediate group, $10^{\gamma}<\varDelta H<30^{\gamma}$, is shown in Fig. 3, in which (a) corresponds to that in the total maximum perid and (b) for total minimum one. In this case, both curves have a common minimum around 9 hr. as mentioned in the case of N-type variation, but the maximum appears around 12hr. in the maximum period, while 21hr. in the minimum one. This type is not only appeared in these total periods, but the similar tendency is also noticeable in each period as shown in (c), showing that this type of the diurnal variation is not accidental.

In order to have more informations about the inequality of the diurnal frequency between the daytime and night, the ratio N'_n/N'_d in the maximum periods is compared with $\overline{\varDelta H}$, the mean value of $\varDelta H$ in each interval in the table 6, where N'_n and N'_d are the mean numbers of SC per three hours in night and day, respectively. It is expressed approximately by the following linear relation, as will be seen in Fig. 4, though the type is so different each others as mentioned above.

Table 6.

Nn	Nd	N'n/N'd	ΔH	$\overline{\Delta H}$	ΔT
9	39	0.38	$\Delta H \leqq 10^{\gamma}$	6.6^{γ}	3.4min.
30	68	0.73	$10<\Delta H<20$	14.5	4.2
26	41	1.05	$20\leqq\Delta H<30$	24.4	3.5
14	15	1.55	$30\leqq\Delta H<40$	34.6	3.7
23	13	2.95	$40\leqq\Delta H$	58.4	3.2

Nn : number in night ; 0, 3 and 21 hr.
Nd : number in daytime ; 6, 9, 12, 15 and 18 hr.

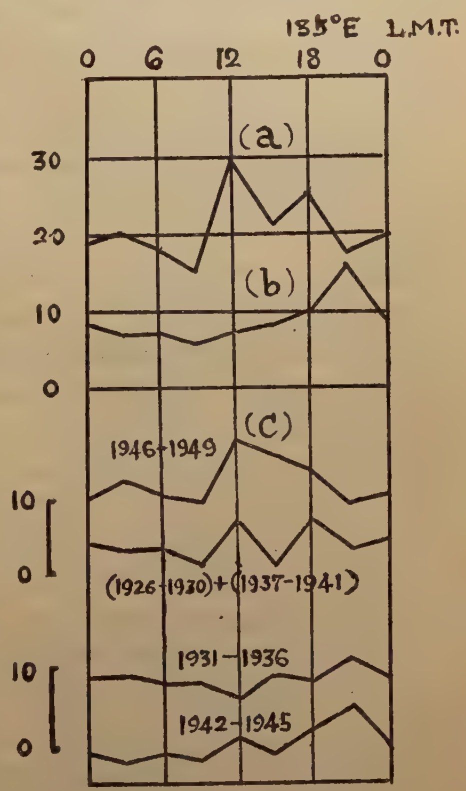

Fig. 3. Number of occurrence of SC.

$$N'_n=4.84\cdot10^{-2}\cdot\overline{\varDelta H}\cdot N'_d.$$

This result, therefore, may be due to the shielding effect of the ionosphere for the external primary field, or to some unknown mechanisms relating to the relative positions of the sun and earth. In this case, it may be worthy to note that the duration

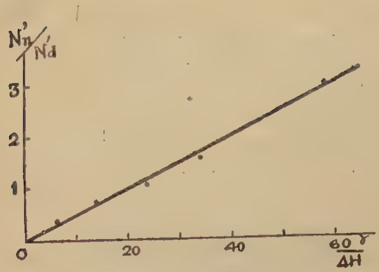

Fig. 4.

$\varDelta T$ of the impulsive change of SC is almost independent of the interval of $\varDelta H$ as will be seen in the last column in the table 6. It is now assumed that (1) the external primary SC giving magnetic effect corresponding to each interval of $\varDelta H$ classified in the table 7 has its corresponding mean value $(\overline{\varDelta H})_0$ when no ionospheres exist. (2) In night hours this is observed as the reduced quantity $q(\overline{\varDelta H})_0$ due to the shielding effects of the ionosphere, and neverthless, the corresponding net number of occurrence in each interval remains approximately constant and equals to $cN'_n \equiv M_n$, where c is a constant. (3) In day hours $(\overline{\varDelta H})_0$ is obsreved as $p(\overline{\varDelta H})_0$, and say $p=1/2 \cdot q$. Then, in the day hours the apparent number M_d and corresponding mean value $(\overline{\varDelta H})_d$ will be grouped as in the table 7. Therefore, the ratio M_n/M_d and total mean value $(\overline{\varDelta H})$ in i-th interval can be calculated in the last two columns by the following expressions, $(M_d)_i = \sum_{j \geqq i} (M_n)_j$ and $(\overline{\varDelta H})_i = \{(M_n)_i \cdot q(\overline{\varDelta H})_{0i} + \sum_{j \geqq i} q(\overline{\varDelta H})_{0j}/2 \cdot (M_n)_j\} / (M_n)_i + \sum_{j \geqq i} (M_n)_j$. The result coincides approximately with the observations in lower values of $\overline{\varDelta H}$ when $q \fallingdotseq 1$. On the other hand, if the uniform primary field, which is situated outside the thin spherical ionosphere with total conductivity K and radius a, varies with respect to time in the form of $F_p = [e^{-\alpha t} - e^{-\lambda \alpha t}]$, then according to the Sugiura's calculation(4), the observable field will be appeared in the form of

$$F_0 = \left[\frac{\beta}{\beta-\alpha} e^{-\alpha t} - \frac{\beta}{\beta-\lambda\alpha} e^{-\lambda\alpha t} + \frac{\alpha\beta(\lambda-1)}{(\beta-\alpha)(\beta-\lambda\alpha)} e^{-\beta t}\right],\ \beta = \frac{3}{4\pi a K},$$

when the inner field due to the presence of the earth is neglected. It is now assumed that α, λ and a are all constants throughout the day and night, while K takes the mean values K_d and K_n in the hours of day and night assumed above, respectively.

Table 7.

No. of interval	Interval	$(\overline{\Delta H})_0$	$(\overline{\Delta H})_n$	Mn	$(\overline{\Delta H})_d$	Md	Mn/Md	$(\overline{\Delta H})$
1	$(\Delta H)_0 \leqq 10^\gamma$	5^γ	$5q^\gamma$	3.0C	2.5q 7.5q	3.0C 10.0C	0.23	6.1q
2	$10 < \Delta H_0 < 20$	15	15q	10.0C	12.5q 17.5q	8.7C 4.7C	0.75	14.5q
3	$20 \leqq \Delta H_0 < 30$	25	25q	8.7C	27.5q	7.7C	1.13	26.8q
4	$30 \leqq \Delta H_0 < 40$	35	35q	4.7C				
5	$40 \geqq \Delta H_0$	55	55q	7.7C				

For the first approximation, if the frst impulse of SC corresponds to the above primary field and is supposed to vary with respect to time in a linear form, above formulae are reduced to $F_p = [\alpha(\lambda-1)t]$ and $F_0 = \alpha\lambda\beta(\lambda-\alpha-\beta+1)t]$. Then we have $F_0 = \frac{\lambda}{\lambda-1}(\lambda-\alpha-\beta+1) \cdot \beta.\ F_p$ and $(F_0)_n/(F_0)_d = K_d/K_n$, where suffixes d and n are meant by day and night. On the other hand, the decreament of the conductivity of the E-layer on the occasion of the total solar eclipse amounts to about fifty percent, and

the ratio $(\overline{f}_{F_2})^2_d/(\overline{f}_{F_2})^2_n$ of the F_2-layer varies approximately in the interval 2–3; for example, 2.4 (1946), 2.2 (1945), and 2.7 (1945) respectively at Kokubunji, Watheroo and Washington. Therefore, it seems to be not so inappropriate to take as $p=\frac{1}{2}q$, and to support qualitatively the argument given above.

For the second-type of SC, we counted the number of occurrence only when the sharp preliminary impulse was accompanied with main SC of the horizontal intensity or declination, and thus it was nine percent. It is very interesting that in Fig. 2 of Ferraro-Parkinson's paper, Greenwitch, Watheroo and Kakioka lie on a smooth curve and similarly Greenwitch, Huancayo, Cheltenham and Tucson make an another curve, both being magnetically synmmetry with respect to the ordinate passing through Honolulu, or very near point to it. Then, it is also interesting to trace the figure in the neighbourhood of the vertex.

4. *Seasonal frequency of occurrence of SC.*

Seasonal frequency curves in the total maximum and minimum sunspot periods are shown respectively in Fig. 5, or table 8. In the sunspot maximum period, we have two principal maxima in equinoxes, and small one in summer, the former distinct seasonal variation being well known for the general geomagnetic activity. In the minimum period, two maxima appear in summer and winter, probably two small maxima in equinoxes. The amplitudes in percent do not so differ each other in both periods, though the number of occurrence is more numerous in maximum period than minimum one.

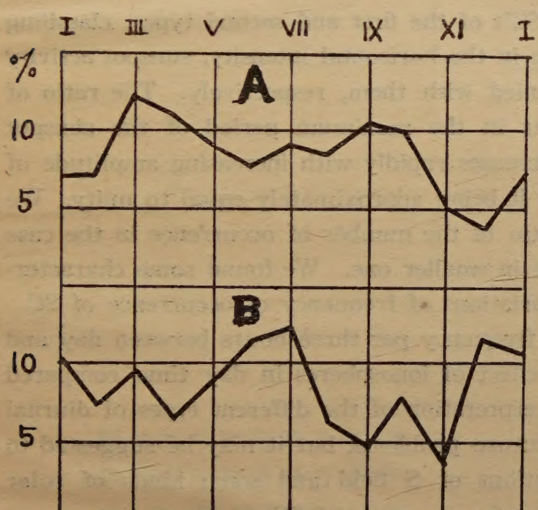

Fig. 5. Seasonal variation.
A: Sunspot max.
B: Sunspot min.

Although the above seasonal variations may be supposed to be more weighted upon the case of $10^\gamma < \Delta H < 30^\gamma$ in which the number is most numerous, similar two different types corresponding to the degree of sunspot activity are also seen in each group of ΔH, in which case of $\Delta H \geqq 30^\gamma$ appears a predominate maximum in spring, sometimes in summer.

Table 8. Seasonal variation of occurrence of SC (percent).

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Max. period	6.9	6.9	12.4	10.9	8.7	7.6	9.1	8.4	10.5	9.8	5.1	3.6
Min. period	10.6	7.1	9.7	6.2	8.8	11.5	12.4	6.2	4.4	8.0	3.5	11.5

It is reasonable, therefore, to expect more or less a flat and smaller amplitude seasonal variation when the frequency is calculated in regardless of the periods of occurrence, as actually shown in Fig. 6. We, moreover, expect an effect of these seasonal characteristics upon the diurnal frequency, for example, if we make two diurnal curves corresponding to equinoxes and (summer) + (winter) in the case of $\Delta H \leqq 10^{\gamma}$, we have similar curves with Fig. 2 (b), a peak maximum around 9 hr. in the former, while 18 hr. in the latter.

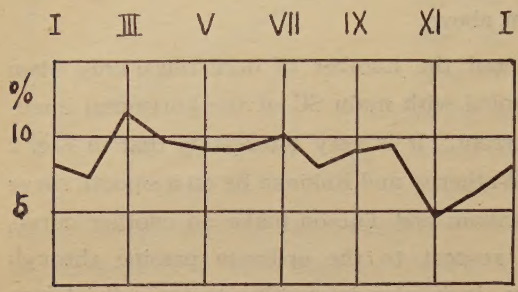

Fig. 6. Seasonal variation in full period.

As a whole, therefore, if the data in the southern hemisphere suports our results on the seasonal variations, the number of occurrence of SC attains its maximum value in local summer in the minimum sunspot period defined above, while in equinoxes in the maximum period.

Summary

We investigated the frequencies of SC's of the first and second types, classifing them in the amplitude of the first impulse in the horizontal intensity, sunspot activity and magnitudes of disturbances accompanied with them, respectively. The ratio of the mean number of occurrence per year in the maximum period of the sunspot activity to that in the minimum period increases rapidly with increasing amplitude of SC, while in the smallest interval $\Delta H \leqq 10^{\gamma}$ it being approximately equal to unity. We have also a similar tendency for the ratio of the number of occurrence in the case of larger amplitude of disturbance to that in smaller one. We found some characteristics of the mean diurnal and seasonal variations of frequency of occurrence of SC.

As a whole, the inequality of mean frequency per three hours between day and night may be due to the larger shielding effect of ionospheres in day time compared with that in night. But the individual interpretation of the different types of diurnal and seasonal variations will be left as future problems, but it may be suggested to pick up anew the possibility of contributions of S field and some kinds of solar radiation as the agency for the production of some parts of SC's.

In conclution the auther wisdes to express his hearty thanks to Dr. S. Imamiti, Director of the Magnetic Observatory at Kakioka for his kind permission to read magnetgraphs and encouragement. His cordial thanks are also due to members of the Observatory for their assistances.

(May 22, 1950)

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昭和25年10月25日 印刷
昭和25年10月31日 發行
第2巻 第2號 定價150圓

編輯兼發行者 日本地球電氣磁氣學會
代表者 長 谷 川 万 吉

印刷者 京都市下京區上鳥羽學校前
田 中 幾 治 郎

賣捌所 丸 善 株 式 會 社 京 都 支 店
丸善株式會社 東京・大阪・名古屋・仙台・福岡

JOURNAL OF GEOMAGNETISM AND GEOELECTRICITY

Vol. II No. 2

1950

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